On right neardomain

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In [1] for exposition of *sharply 2–transitive groups* the concept *neardomain* is introduced as algebraic system with two binary operations $(B_1, 0, \cdot, +, r)$. Until recently it is not known any example of a neardomain which is not a nearfield. In the given work it is offered to loosen neardomain axioms, having left only necessary ones for construction of sharply 2–transitive groups. Let's define the right neardomain as algebraic system $(B_1, 0, v, \cdot, +, -, h, r)$ with operations:

$(+) : B \times B_1 \to B,\ (-) : B \times B_1 \to B,\ (\cdot) : B \times B_1 \to B,$ where $B = B_1 \cup \{1\}$ and

$$v : B_1 \to B_1,\ h : B_1 \times B_1 \to B_1,\ r : B_1 \times B_1 \to B_1,$$

for which axioms are fulfilled
A1. $(\forall x \in B)(\forall y \in B_1)\ (x - y) + y = x$;
A2. $(\forall x \in B)(\forall y \in B_1)\ (x + y) - y = x$;
A3. $(\forall x \in B_1)\ \ x - x = 0$;
A4. $(B_1, \cdot, e)$ is a group with a unit element $e \in B_1$;
A5. $(\forall x \in B)(\forall y, z \in B_1)(\exists\ h(y, z) \in B_1)\ (x + y)z = xh(y, z) + yz$;
A6. $(\forall x \in B)(\forall y, z \in B_1 :\ y + z \neq 0)(\exists\ r(y, z) \in B_1)(x + y) + z = xr(y, z) + (y + z)$;
A7. $(\forall x \in B)(\forall z \in B_1)(\exists\ v(z) \in B_1)\ (x + (0 - z)) + z = xv(z)$.

Let's define a map $L(x) = 0 - x$, then from A1 follows $L(x) + x = 0$. Thus map $L : B_1 \to B_1$ defines left inverse in the right loop.

Lemma. *In the right neardomain the following properties hold:*
1. $(\forall x \in B_1)\ 0x = 0$*;*
2. $h(x, y) = EL(x)L(xy)$*, where* $E(x) = x^{-1}$*;*
3. $r(y, z) = (L(z) - y)^{-1}L(y + z)$*;*
4. $x - z = xv^{-1}(z) + L(z)$*;*
5. $v(z) = EL^2(z)z$*, where* EL *— superposition of transformations* L *and* E*.*

The group $T_2(B)$ of transformations of a set B is called sharply 2–transitive group, if for arbitrary pairs $(x_1, x_2) \neq (y_1, y_2) \in \widehat{B^2}$, where $\widehat{B^2} = B^2 \setminus \{(x, x) | x \in B\}$ there exists an unique element $g \in T_2(B)$ for which the equalities $g(x_1) = y_1$ and $g(x_2) = y_2$ are hold.

Theorem. *Algebraic systems* $(B_1, 0, \varphi, \cdot)$ *and sharply 2–transitive groups* $T_2(B)$ *are rational equivalent.*

The concept *rational equivalence* is introduced by Maltsev A. I. [2].

Let's consider some examples of the right neardomains constructed over a skew field **K**:
1. $x \oplus y = -xa^{-1} + y,\ x \ominus y = -xa + ay,\ r(y, z) = -a^{-1},\ v(z) = a^{-2},\ h(y, z) = z.$
2. $x \oplus y = xy^2 + y,\ x \ominus y = xy^{-2} - y^{-1},\ r(y, z) = y^2 z(z + y)^{-1}(yz + 1),\ h(y, z) = z^{-1}.$

References

[1] *Karzel H.* Inzidenzgruppen I. Lecture Notes by Pieper, I. and Sorensen, K., University of Hamburg (1965), 123-135.

[2] *Maltsev A. I.* Structural performance of some classes of algebras, Doklady of the Academy of Sciences of the USSR, 120, No. 1, 29-32, 1958.